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Output Feedback Control of Discrete-Time Systems in Networked Environments

Chen Peng, Yu-Chu Tian, and Dong Yue

***Abstract*—This paper addresses the problem of output feedback stabilization of control systems in networked environments with quality-of-service (QoS) constraints. The problem is investigated in discrete-time state space using Lyapunov's stability theory and the linear inequality matrix (LMI) technique. A new discrete-time modeling approach is developed to describe NCS with parameter uncertainties and non-ideal network QoS. It integrates network-induced delay, packet dropout and other network behaviors into a unified framework. With this modeling, an improved stability condition, which is dependent on the lower and upper bounds of the equivalent network-induced delay, is established for NCS with norm-bounded parameter uncertainties. It is further extended for output feedback stabilization of NCS with non-ideal QoS. Numerical examples are given to demonstrate the main results of the theoretical development.**

***Index Terms*—Networked control systems; output feedback; maximum allowable equivalent delay bound; stabilization; Quality-of-Service (QoS)**

I. INTRODUCTION

With the rapid development of computer and networking technologies, conventional control system architecture has been evolving to modern networked control, which implements control functionality over data communication networks. Compared with conventional control systems, networked control has a number of advantages, e.g., cost-effectiveness, simplicity in installation and maintenance, and high reliability. Thus, it has received increasing interest in recent years. Introduction to networked control systems (NCS) can be found in [1], [2], [3], [4], [5], [6], [7] and references therein.

An NCS integrates information, communications, and control into a single system in which control loops are closed over computer networks. However, this integration results in some major difficulties in NCS design and implementation [1], [2], [4]. Among many problems in networked control, network-induced delay and data packet dropout are challenging [4], [6], [8], [9], [10]. These problems become more evident and severer when wireless networks are employed and/or when the scheduling of computing and network resources is considered.

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Network-induced delay is time-varying. It affects the accuracy of timing-dependent computing and can degrade the control performance significantly. One of the important issues in NCS analysis and synthesis is to deal with the effect of network-induced delay on NCS control performance.

Data packet dropout results from network traffic congestions and limited network reliability. When a data packet is dropped, complete information of the NCS becomes unavailable. In this case, the controller or actuator has to decide, with incomplete information, what control signals to output.

Because of the problems of network-induced delay and packet dropout, many existing control technologies may become infeasible for specific networked control applications. Therefore, stability analysis and control design of NCS with network-induced delay and packet dropout have received much attention recently [5], [9], [11], [12], [13], [14].

Essential to NCS analysis and design, modeling an NCS with network-induced delay and packet dropout is challenging. Ideally, an NCS should be modeled as a hybrid system and handled using hybrid systems theory, e.g., [9]. However, employing hybrid systems theory will introduce extra complexity into the NCS analysis and design. Thus, simplification of NCS modeling becomes attractive. One of the approaches to simplify NCS modeling is to convert a continuous-time NCS model into a discrete-time one. As in many recent references in the literature, e.g., [15], [16], [17], [18], this paper also adopts the discrete-time modeling technique to describe NCS dynamics. It is worth mentioning that there are occasions where the dynamics of a continuous-time plant model cannot be fully re-constructed from its discrete-time version. Fortunately, most industrial processes under computer control do not belong to this category of systems.

Recently, the control problem of discrete-time systems with time delay was investigated in non-network environments [15], [16]. Hu and Zhu [17], and Nilsson *et al.* [18] considered similar problems in network environments, and developed stochastic optimal controllers based on discrete-time models for the cases when the network-induced delay is larger [17] or shorter [18] than the sampling period. Feng *et al.* [11] adopted the discrete-time modeling technique to analyze the stability and performance of a closed-loop NCS. However, the networked controllers presented in [17], [18] rely on all past information from the starting point, demanding a complicated controller implementation and considerable system memory. They have also ignored the effect of the controller-to-actuator delays on the NCS control performance in order to simplify the system analysis, further limiting the applicability of the controllers to many real-world systems.

The issue of packet dropout has also been addressed in NCS analysis and design in the last few years [5], [8], [13], [14], [18]. Nilsson *et al.* pioneered the modeling of NCS packet dropout as a Markov process [18]. Zhang *et al.* [5] modeled NCS with packet dropout as an asynchronous dynamic system, and derived NCS stability conditions expressed in bilinear matrix inequalities. Yu and colleagues [13] proposed an iterative approach to model NCS with packet dropout as a switching linear system. Although innovative, the iterative approach requires past delay information and ignores variations in time delay. Tian and Levy [8] developed a method of direct compensation for NCS packet dropout without the need of rigorous NCS modeling. Most recently, Xiong and Lam [14] proposed a general framework with a logic zero-order hold (ZOH) to choose the newest control signals.

Despite the above mentioned progress in NCS research, how to analyze and synthesize NCS under constrained network Quality-of-Service (QoS) conditions is still an open problem. This motivates the research of this work for development of output feedback control of dynamic systems with time delay in networked environments. Similar to some existing methods, the methods to be developed in this work employ discrete-time modeling to describe NCS dynamics and consider network-induced delay and packet dropout simultaneously in a unified framework. However, unlike most existing methods, the stability analysis and control design in this work consider non-ideal and constrained network QoS explicitly for trade-offs between the QoS and the NCS control performance.

In the field of computer networking and packet-switched telecommunication networks, QoS refers to a broad set of capabilities and parameters for controlling bandwidth, latency and jitter, and service quality in order to provide a certain level of network performance. It is also used as a quality measure with many alternative definitions. This paper considers latency and jitter, packet loss, and transmission error rate in NCS analysis and synthesis, and leaves other QoS indices to network planning and design.

The main contributions of this work include: (1) A discrete-time modeling approach with parameter uncertainties is developed to describe NCS with non-ideal and constrained network QoS. (2) An improved stability condition and a robust output feedback controller are derived for the modeled NCS with parameter uncertainties. The results are dependent on the lower and upper bounds of network-induced delays. (3) The conditions that give the maximum allowable equivalent delay bound (MAEDB) are obtained for output feedback stabilization of the modeled NCS. This links NCS network QoS and NCS controller design, and consequently leads to a new method for integration and co-design of NCS networks and control.

The paper is organized as follows: Section II proposes a discrete-time modeling approach for NCS with constrained network QoS. Section III establishes the main results of our NCS stability analysis and control synthesis. Examples are given in Section IV. Finally, Section V concludes the paper.

II. Problem Description and NCS Modeling

A. NCS Structure and Basic Assumptions

Consider a continuous-time system

$$\dot{x}(t) = Ax(t) + Bu(t),\ y(t) = Hx(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^m$ and $u(t) \in \mathbb{R}^w$ represent the state, output and control signal of the system, respectively; A, B are constant matrices and H is a nonsingular matrix with appropriate dimension.

For our NCS modeling, the following assumptions are made, which are common in NCS research in the literature.

Assumption 1: Sensors are clock-driven; and all measurement data packets are time stamped.

Assumption 2: The controller is event-driven, and is triggered by the arrival of a measurement data packet. Once control data become available, it sends a control packet to the actuator of the control loop. The control packet carries the time stamps of the measurement data and the control data.

Assumption 3: The actuator is clock-driven and has a logic ZOH. The role of the logic ZOH is to accept a received control packet only if the time stamp of the packet is greater than that of the packet currently stored in the ZOH. A justification of this assumption is given later in Remark 2.

Assumption 4: For any control loop, the measurement data of the plant are transmitted with a single packet in each control period, and so are the control data.

Remark 1: Employing the time stamping technique described in Assumptions 1 and 2, the logic ZOH at the actuator stores the latest control packet. This implies that the logic ZOH discards all control packets but the most recent valid one. The actuator keeps its control signal unchanged until the output of the logic ZOH gets updated to a new value.

Remark 2: Without the ZOH, actuators are normally designed to be event-driven. With the ZOH, either event-driven or clock-driven actuators can be implemented. Clock-driven actuators are assumed in this work for easier control of timing in NCS modeling, stability analysis and controller design.

B. Network-Induced Delay and Packet Dropout

Denote the kth sampling time instant as t_k, i.e.,

$$t_k = kh,\ k \in \mathbb{N} \tag{2}$$

where h is the sampling period, $\mathbb{N}$ stands for the set of non-negative integers. The system samples the value of $y(t)$ at time instant t_k and transmits the sampled data to the NCS network at time instant $t \in [t_k, t_{k+1}]$.

As in [14], for simplicity and without loss of generality, h is normalized to unity. Thus, Eq. (2) is simplified to

$$t_k = k,\ k \in \mathbb{N} \tag{3}$$

Therefore, t_k and k are used interchangeably in this work whenever convenient unless otherwise specified explicitly.

Fig. 1 shows typical time evolution in an NCS, where i_k, i.e., $i_k h$, represents the time stamp of the *latest* valid measurement data packet used for control computation in the kth sampling period. It contains the information of both network-induced delay and packet dropout. For example, in the 10th period ($k = 10$), the control signal available to the actuator is computed from the data sampled in the 7th period ($i_{10} = 7$). This is because: (1) The control packet computed from the sampled data in the 6th period is an out-of-order packet as it arrives at the actuator later than its successive packet derived from the 7th period; and thus it is discarded; (2) For the sampling in the 8th period, either the measurement data packet or the control packet is lost in the transmission, and thus no control signal can be made available to the actuator from this sampling; and (3) The control signal based on the sampling in the 9th period has not arrived yet at the actuator.

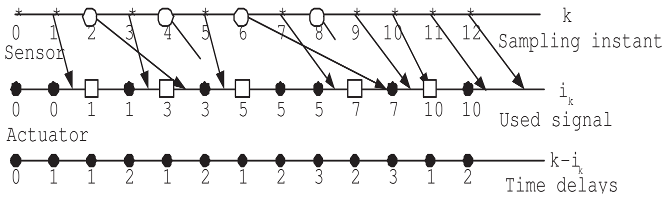

Fig. 1. Packets transmission, dropout, and utilization in an NCS ($*$: packet transmitted; □: control packet used; O: packet lost or discarded).

To model NCS, it is necessary to characterize the equivalent network-induced delay. For sampling instant $k \in \mathbb{N}$, let τ_{i_k} denote the network-induced delay of the latest valid control packet received by the actuator. τ_{i_k} can be expressed as

$$\tau_{i_k} = i_k^d + \eta_{i_k}, \; k \in \mathbb{N}, \; \eta_{i_k} \in [0, 1] \tag{4}$$

where i_k^d and $\eta_{i_k} \in [0, 1]$ are integral and fractional multiples of the sampling period, respectively. For example, in Fig. 1, for $k = 6, 7$ and 8, the latest valid control signal at the actuator is computed from the data sampled in the 5th period. Thus, $i_k^d = 0, i_k = 5$, and $0 < \eta_{i_k} < 1$ for all $k = 6$, 7 and 8.

Moreover, let $i_k^l \in \mathbb{N}$ denote the number of successive sampling periods during which the control signal at the actuator does not get updated in the time interval $[i_k, k]$ due to whatever reasons such as packet dropout and out-of-order transmission. For example, in Fig. 1, for $k = 6$, 7 and 8, we have $i_6^l = 0$, $i_7^l = 1$ and $i_8^l = 2$, respectively. When $i_k^l = 0$, the control signal at the actuator has just been updated right before, or at the beginning of, the current (the kth) sampling period.

Now, define the network allowable equivalent delay $\varrho(k)$ as

$$\varrho(k) = k - i_k, \; k \in \mathbb{N} \tag{5}$$

It is seen from Eq. (5) that $\varrho(k) \in \mathbb{N}$. It follows that

$$\min_{k \in \mathbb{N}} \{\lceil \tau_{i_k} \rceil + i_k^l\} \leq \varrho(k) \leq \max_{k \in \mathbb{N}} \{\lceil \tau_{i_k} \rceil + i_k^l\} \tag{6}$$

where $\lceil \tau_{i_k} \rceil$ gives the smallest integer greater than or equal to τ_{i_k}. Furthermore, define two constant positive integers d_m and d_M, representing the minimum and maximum network allowable equivalent delays, respectively,

$$\begin{aligned} d_m &= \min_{k \in \mathbb{N}} \left\{ \lceil i_k^d + \eta_{i_k} \rceil + i_k^l \right\} \\ d_M &= \max_{k \in \mathbb{N}} \left\{ \lceil i_k^d + \eta_{i_k} \rceil + i_k^l \right\} \end{aligned} \tag{7}$$

Eqs. (4) to (7) justify the following assumption:

Assumption 5: The network allowable equivalent delay $\varrho(k)$ is time-varying and has lower and upper bounds

$$d_m \leq \varrho(k) \leq d_M \tag{8}$$

Remark 3: Network-induced delay $\tau_{i_k} = i_k^d + \eta_{i_k}$ has a non-zero lower bound, which can be larger than a sampling period ($i_k^d \geq 1$) or smaller than a sampling period ($i_k^d = 0$ and $\eta_{i_k} > 0$). The lower bound d_m in Eqs. (7) and (8) can be acquired when the NCS monopolizes the network and CPU resources. Furthermore, it can be inferred from Eq. (8) that a larger network allowable equivalent delay may allow a larger network-induced delay and/or more successive packet losses.

Remark 4: If the control signal at the actuator gets updated in every sampling period, we have $i_k^l = 0 \; \forall \; k \in \mathbb{N}$. In this case, it is seen from Eq. (7) that

$$d_m = \min_{k \in \mathbb{N}} \{\lceil i_k^d + \eta_{i_k} \rceil\}, \; d_M = \max_{k \in \mathbb{N}} \{\lceil i_k^d + \eta_{i_k} \rceil\} \tag{9}$$

Furthermore, if $\tau_{i_k} = i_k^d + \eta_{i_k}$ is a constant τ_0, it can be derived from Eq. (9) that $\varrho(k) = \lceil \tau_0 \rceil = d_m = d_M$. This simplifies the problem to the constant delay case.

C. The Maximum Allowable Equivalent Delay Bound

Now, let us formally introduce the concept of the maximum allowable equivalent delay bound (MAEDB) for NCS.

Definition 1: The MAEDB of an NCS, given by d_M in Eq. (7), is the upper bound of the equivalent delay lumped from the effect of non-ideal and constrained network QoS.

Remark 5: From Definition 1, it is inferred that if $i_k^l > 0$ is allowed, some packet losses can be tolerated in the NCS. This provides a mechanism to schedule network QoS resources for better trade-offs between the network QoS performance and the NCS control performance. For example, if the current level of QoS (e.g., transmission delay) is below the expectation, the NCS may actively drop some packets to improve the QoS performance with some sacrifice of the NCS control performance.

D. Discrete-Time Modeling of Closed-Loop NCS

Under Assumptions 1 to 4, a memoryless output feedback controller with a logic ZOH is now designed for closed-loop control of system (1) over NCS networks. For proportional feedback control with controller gain K, the closed-loop control imposed onto the plant by the actuator in the kth control period is

$$u(k) = Ky(i_k), \; k \in \mathbb{N} \tag{10}$$

Substituting Eq. (5) into Eq. (10) gives

$$u(k) = Ky(k - \varrho(k)), \; k \in \mathbb{N} \tag{11}$$

Then, discretizing the open-loop system in Eq. (1) with consideration of norm-bounded parameter uncertainties and the control signal in Eq. (11), we obtain the following closed-loop system with input delays under networked control

$$x(k+1) = \tilde{F}x(k) + \tilde{G}KHx(k - \varrho(k)) \tag{12}$$

where $\tilde{F}$=$F + \Delta F$, $\tilde{G}$=$G + \Delta G$, F=e^{Ah}, G=$\int_0^h e^{A(h-s)} ds B$. ΔF and ΔG denote parameter uncertainties satisfying

$$[\Delta F, \Delta G] = W \Delta(k) \, [E_1, E_2] \tag{13}$$

where W, E_1 and E_2 are constant matrices of appropriate dimensions; $\Delta(k)$ is an unknown time-varying matrix, which is Lebesgue measurable in k and satisfies $\Delta^T(k)\Delta(k) \leq I$.

III. Main Results

Our main results on NCS stability analysis and control synthesis are summarized in two theorems in this section. The following lemma [19] would be useful in deriving our results.

Lemma 1: Given matrices $\mathcal{F}_1 = \mathcal{F}_1^T$, $\mathcal{F}_2$ and $\mathcal{F}_3$ of appropriate dimensions, we have $\mathcal{F}_1 + \mathcal{F}_3\Delta(k)\mathcal{F}_2 + \mathcal{F}_2^T\Delta^T(k)\mathcal{F}_3^T < 0$ for all $\Delta(k)$ satisfying $\Delta^T(k)\Delta(k) \le I$ if and only if for some $\varepsilon > 0$, $\mathcal{F}_1 + \mathcal{F}_3\varepsilon^{-1}\mathcal{F}_3^T + \mathcal{F}_2^T\varepsilon\mathcal{F}_2 < 0$.

Theorem 1: For given scalars d_m, d_M and matrix K, if there exist matrices $P>0$, $Q>0$ and $R>0$, scalar $\varepsilon>0$, matrices N_i and M_i ($i=1,2,3$) of appropriate dimensions, such that

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & d_M M_1 & E_1^T & \varepsilon N_1 W \\ * & \Pi_{22} & \Pi_{23} & d_M M_2 & H^T K^T E_2^T & \varepsilon N_2 W \\ * & * & \Pi_{33} & d_M M_3 & 0 & \varepsilon N_3 W \\ * & * & * & -d_M R & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \quad (14)$$

is satisfied, where $*$ denotes symmetric elements and

$$\begin{cases} \Pi_{11} = (d_M - d_m + 1)Q + N_1(F - I) \\ \quad\quad +(F-I)^T N_1^T + M_1 + M_1^T \\ \Pi_{12} = N_1 GKH + (F-I)^T N_2^T - M_1 + M_2^T \\ \Pi_{13} = P - N_1 + (F-I)^T N_3^T + M_3^T \\ \Pi_{22} = N_2 GKH + H^T K^T G^T N_2^T - Q - M_2 - M_2^T \\ \Pi_{23} = H^T K^T G^T N_3^T - M_3^T - N_2 \\ \Pi_{33} = P + d_M R - N_3 - N_3^T \end{cases} \quad (15)$$

then the system in Eq. (12) is asymptotically stable.

Proof: First of all, set $\mathfrak{y}(k) = x(k+1) - x(k)$ and choose a Lyapunov functional candidate as

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) \quad (16)$$

where

$$V_1(k) = x^T(k)Px(k),\ V_2(k) = \sum_{k-\varrho(k)}^{k-1} x^T(i)Qx(i),$$

$$V_3(k) = \sum_{\theta=-d_M+2}^{-d_m+1} \sum_{i=k+\theta-1}^{k-1} x^T(i)Qx(i),$$

$$V_4(k) = \sum_{\theta=-d_M}^{-1} \sum_{i=k+\theta}^{k-1} \mathfrak{y}^T(i)R\mathfrak{y}(i), P>0, Q>0, R>0.$$

Then, taking the increment of Eq. (16), we have

$$\Delta V(k) = \sum_{i=1}^{4} \Delta V_i(k) \quad (17)$$

Each of $\Delta V_1(k)$ to $\Delta V_4(k)$ in Eq. (17) is derived as follows:

$$\Delta V_1(k) = 2x^T(k)P\mathfrak{y}(k) + \mathfrak{y}^T(k)P\mathfrak{y}(k) \quad (18)$$

$$\Delta V_2(k) = x^T(k)Qx(k) - x^T(k-\varrho(k))Qx(k-\varrho(k)) + \sum_{i=k-\varrho(k+1)+1}^{k-1} x^T(i)Qx(i) - \sum_{i=k-\varrho(k)+1}^{k-1} x^T(i)Qx(i) \quad (19)$$

Considering Eq. (8), we obtain

$$\begin{aligned} & \sum_{i=k-\varrho(k+1)+1}^{k-1} x^T(i)Qx(i) \\ &= \sum_{i=k-d_m+1}^{k-1} x^T(i)Qx(i) + \sum_{i=k-\varrho(k+1)+1}^{k-d_m} x^T(i)Qx(i) \\ &\le \sum_{i=k-\varrho(k)+1}^{k-1} x^T(i)Qx(i) + \sum_{i=k-d_M+1}^{k-d_m} x^T(i)Qx(i) \quad (20) \end{aligned}$$

It follows from Eqs. (19) and (20) that

$$\begin{aligned} \Delta V_2(k) \le\ & x^T(k)Qx(k) + \sum_{i=k-d_M+1}^{k-d_m} x^T(i)Qx(i) \\ & - x^T(k-\varrho(k))Qx(k-\varrho(k)) \quad (21) \end{aligned}$$

We also have $\Delta V_3(k)$ and $\Delta V_4(k)$ as

$$\begin{aligned} \Delta V_3(k) =\ & (d_M - d_m)x^T(k)Qx(k) \\ & - \sum_{i=k-d_M+1}^{k-d_m} x^T(i)Qx(i) \quad (22) \end{aligned}$$

$$\Delta V_4(k) = d_M \mathfrak{y}^T(i)R\mathfrak{y}(i) - \sum_{i=k-d_M}^{k-1} \mathfrak{y}^T(i)R\mathfrak{y}(i) \quad (23)$$

For deleting the accumulative term in (23),the following two equations are introduced:

$$2\zeta^T(k)M^T[x(k) - x(k-\varrho(k)) - \sum_{i=k-\varrho(k)}^{k-1} \mathfrak{y}(i)] = 0 \quad (24)$$

$$2\zeta^T(k)N^T\left[(\tilde{F} - I)x(k) - \mathfrak{y}(k) + \tilde{G}KHx(k-\varrho(k))\right] = 0 \quad (25)$$

where

$$\zeta^T(k) = \left[x^T(k), x^T(k-\varrho(k)), \mathfrak{y}^T(k)\right] \quad (26)$$

$N = [N_1^T, N_2^T, N_3^T]$, $M = [M_1^T, M_2^T, M_3^T]$, M_i and N_i ($i = 1,2,3$) are arbitrary matrices of appropriate dimensions.

From Eqs. (18) through to (23) and Eqs. (24) and (25), $\Delta V(k)$ in Eq. (17) can be magnified as

$$\begin{aligned} \Delta V(k) \le\ & \zeta^T(k)\Pi\zeta(k) - \sum_{i=k-d_M}^{k-1} \mathfrak{y}^T(i)R\mathfrak{y}(i) \\ & - 2\zeta^T(k)M^T \sum_{i=k-\varrho(k)}^{k-1} \mathfrak{y}(i) \quad (27) \end{aligned}$$

where

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ * & \Pi_{22} & \Pi_{23} \\ * & * & \Pi_{33} \end{bmatrix} + \mathbb{E}\Delta^T(k)\mathbb{W} + \mathbb{W}^T\Delta(k)\mathbb{E}^T$$

$$\mathbb{E} = \begin{bmatrix} E_1^T \\ H^T K^T E_2^T \\ 0 \end{bmatrix},\ \mathbb{W}^T = \begin{bmatrix} N_1 W \\ N_2 W \\ N_3 W \end{bmatrix}$$

Now, we prove that $\Delta V(k) < 0$. It follows from Definition 1 and Eq. (8) that

$$-\sum_{k-d_M}^{k-1} \mathfrak{y}^T(i)R\mathfrak{y}(i) \leq -\sum_{k-\varrho(k)}^{k-1} \mathfrak{y}^T(i)R\mathfrak{y}(i) \quad (28)$$

It is also seen that for any $R > 0$

$$\begin{aligned} &-2\zeta^T(k)M^T\left[\sum_{i=k-\varrho(k)}^{k-1}\mathfrak{y}(i)\right] \\ &\leq d_M\zeta^T(k)M^TR^{-1}M\zeta(k) + \sum_{i=k-\varrho(k)}^{k-1}\mathfrak{y}^T(i)R\mathfrak{y}(i) \end{aligned} \quad (29)$$

From Eqs. (27), (28) and (29), we obtain

$$\Delta V(k) \leq \zeta^T(k)\ (\Pi + d_M M^T R^{-1} M)\ \zeta(k) \quad (30)$$

It is known from Lemma 1 that there exists $\varepsilon > 0$ such that

$$\mathbb{E}\Delta^T(k)\mathbb{W} + \mathbb{W}^T\Delta(k)\mathbb{E}^T \leq \varepsilon^{-1}\mathbb{E}\mathbb{E}^T + \varepsilon\mathbb{W}^T\mathbb{W} \quad (31)$$

From Eqs. (30) and (31) and using Schur's complement, the condition in Eq. (14) guarantees that $\Delta V(k)$ in Eq. (30) satisfies $\Delta V(k) < 0$ for all non-zero $\zeta(k)$. This implies that the condition in Eq. (14) guarantees the asymptotical stability of the system in Eq. (12) for all $\varrho(k)$ satisfying $d_m \leq \varrho(k) \leq d_M$. This completes the proof. ■

Theorem 2: For given scalars d_m, d_M and λ_2, λ_3, the norm-bounded uncertain system in Eq. (12) is robustly asymptotically stable with feedback gain matrix $K = YX^{-T}H^{-1}$ if there exist matrices $\tilde{P} > 0$, $\tilde{Q} > 0$ and $\tilde{R} > 0$, a non-singular matrix X, a scalar $\varepsilon > 0$, and matrices Y and $\tilde{M}_i$ $(i = 1, 2, 3)$ of appropriate dimensions, such that

$$\begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & d_M\tilde{M}_1 & XE_1^T & \varepsilon W \\ * & \bar{\Pi}_{22} & \bar{\Pi}_{23} & d_M\tilde{M}_2 & Y^TE_2^T & \varepsilon\lambda_2 W \\ * & * & \bar{\Pi}_{33} & d_M\tilde{M}_3 & 0 & \varepsilon\lambda_3 W \\ * & * & * & -d_M\tilde{R} & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \quad (32)$$

is satisfied, where

$$\begin{cases} \bar{\Pi}_{11} = (d_M - d_m + 1)\tilde{Q} + (F - I)X^T \\ \qquad + X(F - I)^T + \tilde{M}_1 + \tilde{M}_1^T, \\ \bar{\Pi}_{12} = GY + \lambda_2 X(F - I)^T - \tilde{M}_1 + \tilde{M}_2^T, \\ \bar{\Pi}_{13} = \tilde{P} - X^T + \lambda_3 X(F - I)^T + \tilde{M}_3^T, \\ \bar{\Pi}_{22} = -\tilde{Q} + \lambda_2 GY + \lambda_2 Y^T G^T - \tilde{M}_2^T - \tilde{M}_2, \\ \bar{\Pi}_{23} = -\lambda_2 X^T + \lambda_3 Y^T G^T - \tilde{M}_3^T, \\ \bar{\Pi}_{33} = \tilde{P} + d_M\tilde{R} - \lambda_3 X^T - \lambda_3 X. \end{cases} \quad (33)$$

Proof: First of all, Theorem 1 guarantees the asymptotical stability of the system in Eqs. (12) and (13) under the condition of Eq. (14). Then, if we show that the condition of Eq. (32) can be derived from Eq. (14), the theorem is proven to be true.

Denote $N = X^{-1}$, $N_1 = N$, $N_2 = \lambda_2 N$ and $N_3 = \lambda_3 N$; and also define $\tilde{P} = XPX^T$, $\tilde{Q} = XQX^T$, $\tilde{R} = XRX^T$, $\tilde{M}_i = XM_iX^T$ $(i = 1, 2, 3)$ and $Y = KHX^T$. Pre- and post-multiplying Eq. (14) with $diag(X, X, X, X, I, I)$ and its transpose, respectively, yields Eq. (32).

Furthermore, when Eq. (32) holds, we have $\bar{\Pi}_{33} = \tilde{P} + d_M\tilde{R} - \lambda_3 X^T - \lambda_3 X < 0$. From the assumption of Theorem 2, $\tilde{P} + d_M\tilde{R} > 0$. Assuming $\lambda_3 > 0$, we can obtain $X^T + X > 0$, where X is non-singular. This completes the proof. ■

Remark 6: For a given discrete-time NCS in Eq. (12), the maximum allowable equivalent delay d_M can be derived from Theorem 1 or 2. It is seen from Eq. (8) that the resulting d_M is the MAEDB, into which the effect of network induced-delay, packet dropout and other QoS parameters on the NCS control performance is lumped. Thus, the MAEDB can help relate the control performance to the network QoS, and can be used to schedule NCS resources through adjusting one or more QoS parameters for a better trade-off between the control performance and the network QoS. For example, actively dropping some packets can be a way of reducing traffic congestions and network-induced delay in heavy traffic conditions.

Remark 7: It is seen from the proof of Theorem 2 that in order to derive the condition in Eq. (32) from Eq. (14), we have set $N_1 = N$, $N_2 = \lambda_2 N$ and $N_3 = \lambda_3 N$. The resulting condition in Eq. (32) becomes tighter than Eq. (14). This implies that the result derived from Eq. (14) is generally less conservative than that obtained from Eq. (32). However, Eq. (32) is directly applicable to feedback controller design.

The results presented in Theorem 2 are related to given scalars d_M, λ_2 and λ_3. This means that direct computation of optimal d_M and K values from Theorem 2 becomes infeasible. Therefore, the following algorithm is developed.

Algorithm 1: Iterative computation of d_M and K.

Step 1. Give α_i and β_i as the upper and lower bounds of λ_i, respectively, and ζ_i as the step increments $(i = 2, 3)$; let $\lambda_i := \alpha_i$. Set initial $d_M(0) := 0$ and $K_0 := 0$.

Step 2. Use the LMI toolbox to compute $d_M(\lambda_i)$ and $K := YX^{-T}H^{-1}$ subject to Eq. (32). If $d_M(\lambda_i) > d_M(0)$, the result is improved in this iteration and thus update $d_M(0) := d_M(\lambda_i)$ and $K_0 := K$.

Step 3. Update $\lambda_3 := \lambda_3 + \zeta_3$. If λ_3 is still in the search range, i.e., $\lambda_3 \in [\alpha_3, \beta_3]$, then go to Step 2.

Step 4. Update $\lambda_2 := \lambda_2 + \zeta_2$. If λ_2 is still in the range $\lambda_2 \in [\alpha_2, \beta_2]$, reset $\lambda_3 := \alpha_3$ and go to Step 2.

Step 5. Output $d_M(0)$ and K_0.

Step 6. With the obtained K_0 from Step 5, use the LMI toolbox to find d_M subject to Eq.(14).

Step 7. Output d_M, and exit with success.

IV. NUMERICAL EXAMPLES

Example 1: Consider an inverted pendulum governed by

$$\dot{z}(t) = \begin{bmatrix} 0 & 1 \\ \frac{3(M+m)g}{l(4M+m)} & 0 \end{bmatrix} z(t) + \begin{bmatrix} 0 \\ -\frac{3}{l(4M+m)} \end{bmatrix} u(t) \quad (34)$$

with M = 8.0kg, m = 2.0kg, l = 0.5m, g = 9.8m/s^2 and h = 30ms. Discretizing the continuous-time plant model in Eq. (34) gives the following discrete-time plant model [16]:

$$x(k+1) = \begin{bmatrix} 1.0087, \ 0.0301 \\ 0.5202, \ 1.0087 \end{bmatrix} x(k) + \begin{bmatrix} -0.0001 \\ 0.0053 \end{bmatrix} u(k) \quad (35)$$

Assume the lower delay bound $d_m = 1$ (i.e., $d_m h = 30$ms). Applying Theorem 3 of [16], we have obtained $d_M = 3$ (i.e., $d_M h = 90$ms). In comparison, using Theorem 2 of this paper, we have found that the system is always stable for any delay less than $d_M = 5$ (i.e., $d_M h = 150$ms) with given $\lambda_2 = -0.8$ and $\lambda_3 = 4.2$. A networked output feedback controller that stabilizes the system is given by

$$u(k) = [106.5970, 33.8599]\, x(k - \varrho(k)) \tag{36}$$

Now, let us investigate the scenario where network-induced delay changes randomly in the range $1 \le \varrho(k) \le 5$. Assume that this random delay has a Gaussian distribution. The initial condition of the system is arbitrarily chosen as $x(0) = [2, -1]^T$. Applying the networked controller in Eq. (36), we obtain the closed-loop system dynamics in Fig. 2. It is seen from this figure that the system is stabilized.

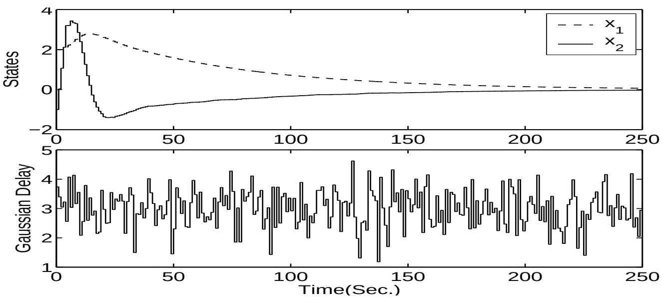

Fig. 2. System responses with Gaussian distribution of $\varrho(k) \in [1, 5]$.

Example 2: Consider the closed-loop NCS in Eq. (12) with norm-bounded parameter uncertainties in Eq. (13). It is discretized from the open-loop system in Eq. (1) and the feedback controller in Eq. (11). The settings of the system are: $F = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.9 \end{bmatrix}$, $G = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $W = \begin{bmatrix} \bar{\alpha} \\ 0 \end{bmatrix}$, $E_1 = [1, 0]$, $E_2 = [0, 0]$, $\Delta(k) = \alpha(k)/\bar{\alpha}$, where $|\alpha(k)| \le \bar{\alpha}$.

Assume that both the lower and upper bounds of the delay $\varrho(k)$ are known. Our purpose is to determine the maximum value of $\bar{\alpha}$ such that the system in Eq. (12) with the uncertainties in Eq. (13) is asymptotically stable.

Table I tabulates our simulation results from Theorem 2 of this paper with $\lambda_2 = 0.1$ and $\lambda_3 = 4.3$. For comparison, the methods from [15], [16] are also simulated under the same conditions; and the results are listed in Table I as well.

It is seen from Table I that the results derived from this paper are less conservative than those from [15], [16]. Some improvements are significant, e.g., 20% and 33% over [16] for $\varrho(k) \in [2, 7]$ and $\varrho(k) \in [2, 8]$, respectively.

TABLE I
THE MAXIMUM VALUE OF $\bar{\alpha}$.

Method	$\varrho(k) \in [3, 5]$	$\varrho(k) \in [2, 7]$	$\varrho(k) \in [2, 8]$
Reference [15]	0.1650	0.0830	infeasible
Reference [16]	0.2405	0.1901	0.1667
Theorem 2 of this work	0.2467	0.2286	0.2231
Improvement over [16]	$> 2.5\%$	$> 20\%$	$> 33\%$

V. CONCLUSION

A new discrete-time modeling approach has been developed in this paper to describe NCS with network-induced delay and packet dropout. With this modeling, a less conservative stability condition has been established, which is dependent on the lower and upper bounds of the equivalent network-induced delay, to guarantee the asymptotic stability of the NCS with parameter uncertainties. From this condition, the maximum allowable equivalent delay bound can be derived, which links network QoS and controller design and consequently suggests a new method for integrated design of NCS networks and control. The stability analysis results have been further applied to output feedback stabilization of NCS.

REFERENCES

[1] P. Antsakls and J. Baillieul, "Guest editorial: Specia issue on networked control systems," *IEEE Trans. Automat. Control*, vol. 49, no. 9, pp. 1421–1423, Sep. 2004.

[2] ——, "Guest editorial: Special issue on technology of networked control systems," *Proc. of the IEEE*, vol. 95, no. 1, pp. 5–8, Jan. 2007.

[3] F. L. Lian, J. R. Moyne, and D. M. Tilbury, "Network design considera-tion for distributed control systems," *IEEE Trans. Control Syst. Technol.*, vol. 10, no. 2, pp. 297–307, Mar. 2002.

[4] Y.-C. Tian and D. Levy, "Dealing with network complexity in real-time networked control," *Int. J. Computer Math.*, vol. 85, no. 8, pp. 1235–1253, Aug. 2008.

[5] W. Zhang, M. S. Branicky, and S. M. Phillips, "Stability of networked control systems," *IEEE Control Syst. Mag.*, vol. 21, no. 1, pp. 84–99, Feb. 2001.

[6] D. Yue, E. G. Tian, Z. D. Wang, and J. Lam, "Stabilization of Systems With Probabilistic Interval Input Delays and Its Applications to Networked Control Systems ," *IEEE Trans. Syst., Man & Cybern., Part A: System and Humans*, vol. 39, no. 4, pp. 939–945, Jul. 2009.

[7] H. G. Zhang, M. Li, J. Yang, and D. D. Yang, "Fuzzy Model-Based Robust Networked Control for a Class of Nonlinear Systems," *IEEE Trans. Syst., Man & Cybern., Part A: System and Humans*, vol. 39, no. 2, pp. 437–447 , Mar. 2009.

[8] Y.-C. Tian and D. Levy, "Compensation for control packet dropout in networked control systems," *Information Sciences*, vol. 178, no. 5, pp. 1263–1278, Mar. 2008.

[9] D. Yue, Q. L. Han, and C. Peng, "State feedback controller design of networked control systems," *IEEE Trans. Circ. & Syst. II: Express Briefs*, vol. 51, no. 11, pp. 640–644, Nov. 2004.

[10] C. Peng and Y.-C. Tian, "State feedback controller design of networked control systems with interval time-varying delay and nonlinearity," *Int. J. Robust Nonlinear Control*, vol. 18, no. 12, pp. 1285–1301, Oct. 2008.

[11] F.-L. Lian, J. R. Moyne, and D. M. Tilbury, "Modelling and optimal controller design of networked control systems with multiple delays," *Int. J. Control*, vol. 76, no. 6, pp. 591–606, 2003.

[12] Y. K. Lin, "Reliability evaluation for an information network with node failure under cost constraint," *IEEE Trans. Syst., Man & Cybern., Part A: System and Humans*, vol. 39, no. 2, pp. 180–188, Mar. 2007.

[13] M. Yu, L. Wang, T. G. Chu, , and G. M. Xie, "Stabilization of networked control systems with data dropout and network delays via switching system approach," in *Proc. the 43rd IEEE Conf. on Decision & Control*, Atlantis, Paradise Island, Bahamas, 2004, pp. 3539–3544.

[14] J. L. Xiong and J. Lam, "Stabilization of networked control systems with a logic ZOH," *IEEE Trans. Automat. Control*, vol. 54, no. 2, pp. 358–363, Feb. 2009.

[15] H. Gao, J. Lam, C. Wang, and Y. Wang, "Delay dependent out-feedback stabilisation of discrete-time systems with time-varying state delay," *IEE Proc.: Control Theory & Appl.*, vol. 151, no. 6, pp. 691–698, Nov. 2004.

[16] H. Gao and T. W. Chen, "New results on stability of discrete-time systems with time-varying state delay," *IEEE Trans. Automat. Control*, vol. 52, no. 2, pp. 328–334, Feb. 2007.

[17] S. S. Hu and Q. X. Zhu, "Stochastic optimal control and analysis of stability of networked control systems with long delay," *Automatica*, vol. 39, no. 11, pp. 1877–1884, Nov. 2003.

[18] J. Nilsson, B. Bernhardsson, and B. Wittenmark, "Stochastic analysis and control of real-time systems with random time delay," *Automatica*, vol. 34, no. 1, pp. 57–64, Jan. 1998.

[19] L. Xie, M. Fu, and C. De Souza, "H_∞ control and quadratic stabilization of system with parameter uncertainty via output feedback," *IEEE Trans. Autom. Control*, vol. 37, no. 8, pp. 1253–1257, Aug. 1992.